Bachelor of Science (B.Sc.) Semester—I Examination

MATHEMATICS

(M_1–Algebra & Trigonometry)

Compulsory Paper—I

Time—Three Hours] [Maximum Marks—60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question No. 1 to 4 have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Find non-singular matrices P and Q, so that PAQ is in the normal form, where

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}.$$ 6

(B) Show that the equations $x + 2y - z = 3$, $3x - y + 2z = 1$, $2x - 2y + 3z = 2$, $x - y + z = -1$ are consistent and solve them. 6

OR

(C) Find the eigen values and corresponding eigen vectors of the matrix :

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$ 6

(D) Show that the matrix $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ satisfies Cayley Hamilton Theorem. 6

UNIT—II

2. (A) Find the condition that the roots of the equation $x^3 - px^2 + qx - r = 0$ be in geometrical progression and hence solve the equation $x^3 + 14x^2 + 56x + 64 = 0$. 6

(B) Solve the reciprocal equation :

$$x^4 - 5x^3 + 8x^2 - 5x + 1 = 0.$$ 6

OR

(C) Solve $x^3 - 21x - 344 = 0$ by Cardon's method. 6

(D) Solve $x^4 + 4x^3 + 12x^2 - 8x + 95 = 0$ by Ferrari's method. 6

UNIT—III

3. (A) Prove that $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$, if n is a positive integer and negative integer. 6

(B) Prove that :

$$(a+ib)^{m/n} + (a-ib)^{m/n} = 2(a^2+b^2)^{m/2n}\cos\left[\frac{m}{n}\left(\tan^{-1}\frac{b}{a}\right)\right].$$

6

OR

(C) If $\cosh(a + i\beta) = x + iy$, prove that :

$$\frac{x^2}{\cosh^2\alpha} + \frac{y^2}{\sinh^2\alpha} = 1 \text{ and } \frac{x^2}{\cos^2\beta} + \frac{y^2}{\sin^2\beta} = 1.$$

6

(D) Separate into real and imaginary parts :

(i) $\cot(\alpha + i\beta)$ and

(ii) $\log(x + iy)$. 6

UNIT—IV

4. (A) Let $G = (G, o)$ be a group, then prove that :

(i) $(a^{-1})^{-1} = a$ for all a in G

(ii) $(a \circ b)^{-1} = b^{-1} \circ a^{-1}$ for all a, b in G. 6

(B) Prove that the set $G = \{1, -1, i, -i\}$ is an abelian multiplicative finite group of order 4. 6

OR

(C) Prove that the order of a subgroup of a finite group is a divisor of the order of the group. 6

(D) Prove that the union of two subgroups is a subgroup if and only if one is contained in the other. 6

Question—V

5. (A) Find the rank of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \end{pmatrix}$ by reducing to normal form. 1½

(B) For $A^2 - 2A - I = 0$, find A^{-1} if $A = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$. 1½

(C) By using Descarte's rule of signs, find the nature of the roots of the equation $x^3 - 2x^2 + 3x - 7 = 0$. 1½

(D) If α, β, γ be roots of the cubic $x^3 + px^2 + qx + r = 0$ then calculate the value of $\Sigma\alpha^2$. 1½

(E) Find all the values of $(1)^{1/3}$. 1½

(F) Prove that $\log(i) = \frac{1}{2}\pi i$. 1½

(G) If $f = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ be two permutations of degree 3. Find fg and gf. Show that $fg \neq gf$. 1½

(H) Determine whether the permutation $f = (1\ 2\ 3\ 4\ 5)\ (1\ 2\ 3)\ (4\ 5)$ is odd or even. 1½